3D entropy wave generation

1. Input equation

$$T(x,y,z,t) \sim A\left\{\sum_i [S_1(\xi_i)\Delta\xi_i]^{\frac{1}{2}}\cos\left[\xi_i\left(\mathrm{y}-v_i\mathrm{t}+\frac{v_i x}{\bar{u}}\right)+\phi_i\right]\right\}$$

$$\left\{\sum_j \left[S_2(\eta_j)\Delta\eta_j\right]^{1/2}\cos\left[\eta_j\left(\mathrm{z}-W_j\mathrm{t}+\frac{W_j x}{\bar{u}}\right)+\psi_i\right]\right\}$$

$$\left\{\sum_k [S_3(\zeta_k)\Delta\zeta_k]^{1/2}\cos\left[\zeta_k\left(-\mathrm{t}+\frac{x}{\bar{u}}\right)+\Theta_k\right]\right\}$$

Time-correlation function is

$$R(\tau) = \overline{T^2}e^{-\frac{\tau^2\sigma^2}{4\ln(2)}}\cos(\omega_0\tau)$$

$\sigma = \frac{\omega_0}{2}$, $\omega_0 = 2\pi\bar{u}$, $\bar{u} = 0.3$, $\overline{T^2} = 1$ and the corresponding spectra are

$$S(\omega) = \overline{T^2}\left(\frac{\ln(2)}{\pi}\right)^{1/2}\frac{1}{2\sigma}e^{-\ln(2)\left[\frac{\omega-\omega_0}{\sigma}\right]^2} + e^{-\ln(2)\left[\frac{\omega+\omega_0}{\sigma}\right]^2}$$

S_1 and S_2 are the same, and is Fourier transform of

$$\overline{T(x,y',z,t)T(x,y'',z,t)} = \overline{T^2}e^{-\ln(2)\left[\frac{y'-y''}{b}\right]^2}\ ,\ b = 0.75$$

$$S_1(\xi) = \frac{1}{2\pi}\left(\frac{\pi}{\ln(2)}\right)^{1/2} be^{-\frac{[\xi b]^2}{4\ln(2)}}$$

$S_3(\zeta)$ is the Fourier transform of $R_3(\tau)$

$$R_3(\tau) = \frac{\overline{T(x,y,z,t)T(x,y,z,t+\tau)}}{\overline{T(x,y',z,t)T(x,y'',z,t)}}\frac{\overline{T^2(x,y,z,t)}}{\overline{T(x,y,z',t)T(x,y,z'',t)}}$$

in which replace $(y-y')$ by $|V|\tau$, replace $(z-z')$ by $|W|\tau$. Since $\Delta\tau = 0.05$, $\Delta y = \Delta z = 0.1/7$, in order to have appropriate $R_3(\tau)$ and thus facilitate the computation, we firstly let $|V| = |W| = 2/7$, in such scenario, $\frac{\Delta y}{|V|} = \frac{\Delta z}{|W|} = 0.05$, which is exactly $\Delta\tau$.

$$S_3(\zeta) = \frac{1}{2\pi}\left(\frac{\pi}{\frac{\sigma^2}{4\ln(2)}-\frac{V^2+W^2}{b^2}\ln(2)}\right)^{1/2} e^{-\frac{\omega_0^2+\zeta^2}{4\left[\frac{\sigma^2}{4\ln(2)}-\frac{V^2+W^2}{b^2}\ln(2)\right]}}\cosh\left[\frac{\omega_0\zeta}{2\left[\frac{\sigma^2}{4\ln(2)}-\frac{V^2+W^2}{b^2}\ln(2)\right]}\right]$$

In which we can validate

$$\frac{\sigma^2}{4\ln(2)}-\frac{V^2+W^2}{b^2}\ln(2) > 0$$

And thus spectra $S_3(\zeta)$ is feasible.

2. Parameters

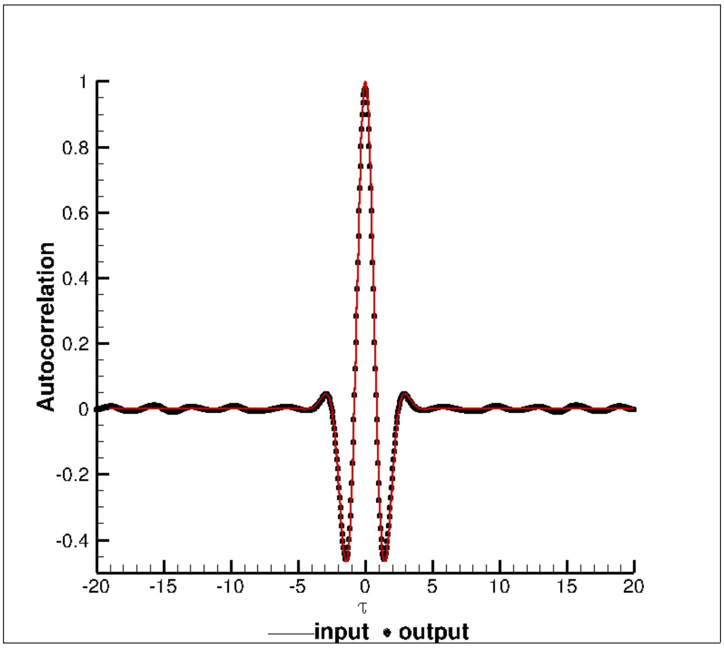
All of these three spectra are cut to have $n = 1500$. For S_1 and S_2, $\xi_{max} = \xi_{n=1500} = 4\omega_0$, then $S_1(\xi_{max}) = 9.8 \times 10^{-6} max(S_1(\xi)) = 9.8 \times 10^{-6} S_1(0)$, which is considered to be small enough. For S_3, $\zeta_{max} = \zeta_{n=1500} = 2.7\omega_0$, then $S_3(\zeta_{max}) \approx 1.8 \times 10^{-6} max(S_3(\zeta))$, which is also considered to be adequate small. Since

$$\omega_j = \frac{\Delta\omega_1}{2} r^{j-1} + \Delta\omega_1 \frac{r^{j-1} - 1}{r - 1}$$

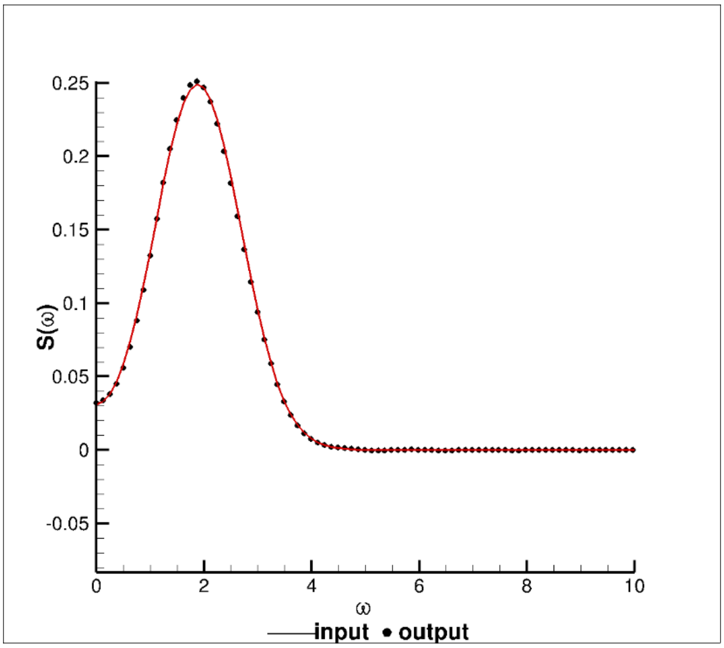
In discretizing S_1 and S_2, we set $r=1.0005$, then $\Delta\xi_1 = \frac{\xi_{\max}}{2232.08}$, in discretizing S_3, we set $r=1.0006$, then $\Delta\zeta_1 = \frac{\zeta_{\max}}{2430.03}$.

Similar to the 2D test case analysis, In choosing time-related variables, we choose $\Delta t = 0.05$, $T_{total} = 60{,}000$, and in choosing space-related parameters, we choose $\Delta x = 0.1/7$, $L = 5.0$ and there are 351 points in x direction, $\Delta x = \Delta z = 0.1/7$, $W = H = 7.2$ and there are 1009 points in y and z directions.

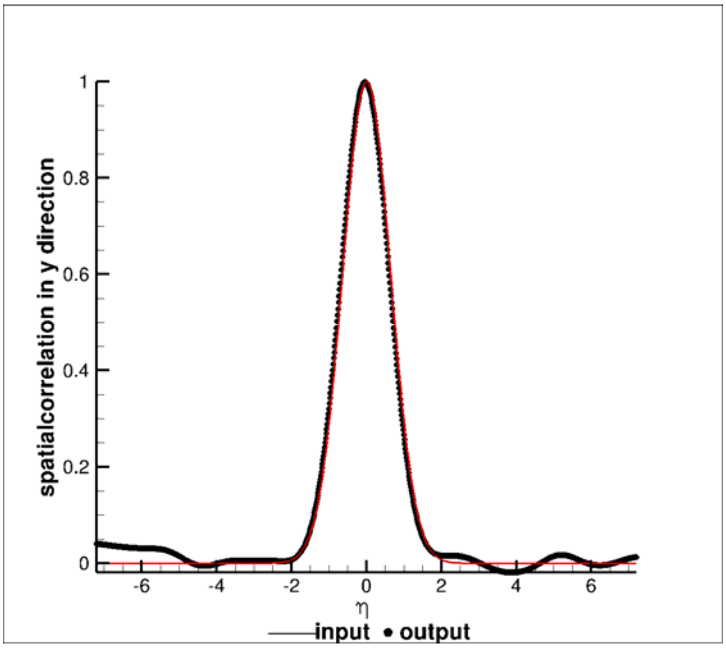
3. Test results



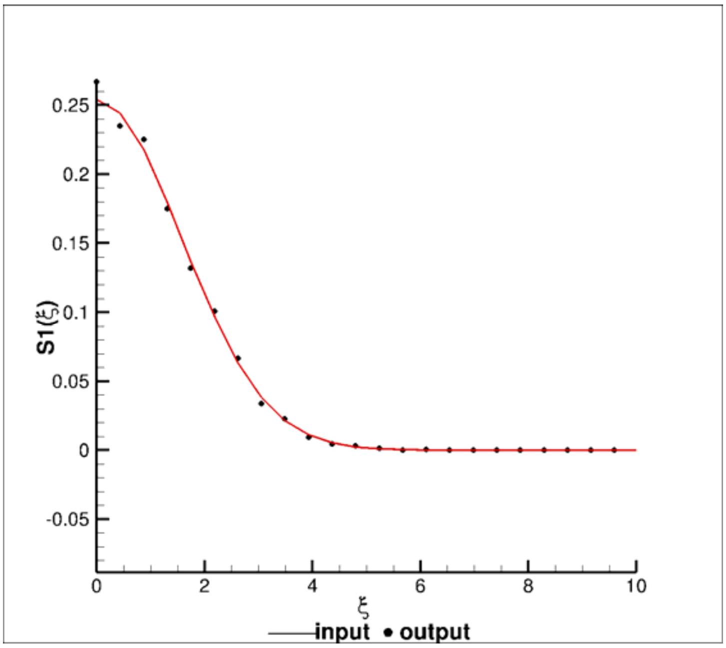
Time-correlation $R(\tau)$



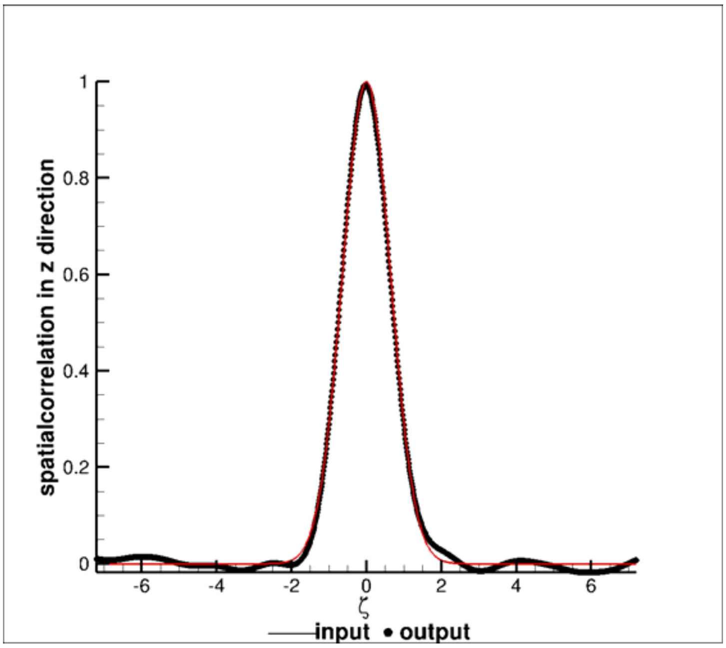
Spectra $S(\omega)$



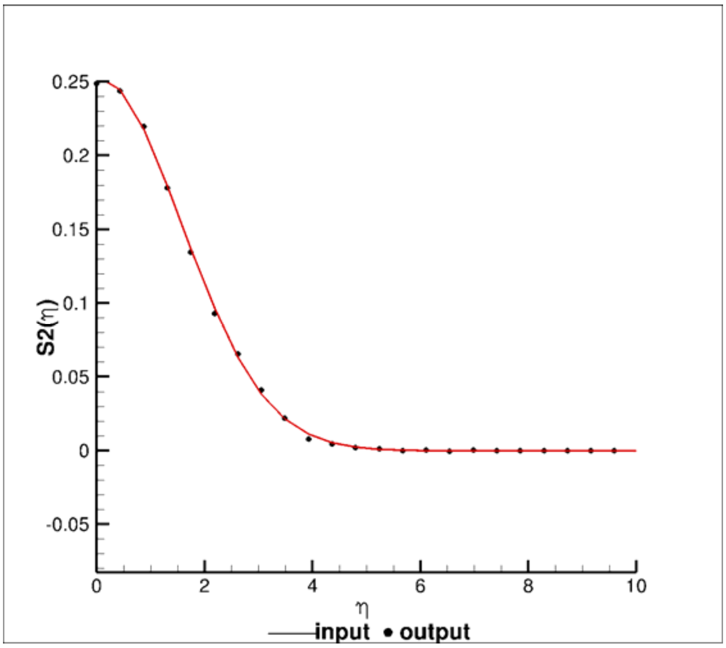
Spatial-correlation in x direction $\overline{T(x, y', z, t)T(x, y'', z, t)}$



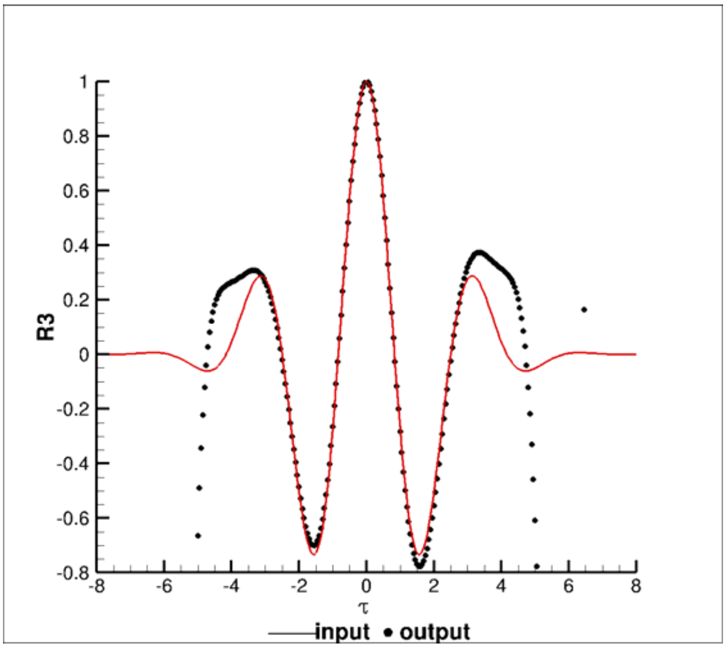
Spectra $S_1(\xi)$



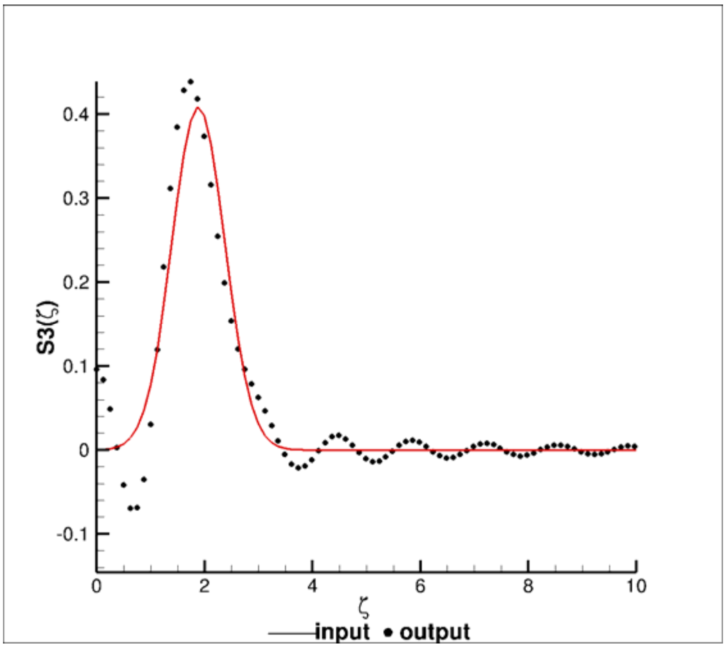
Spatial-correlation in z direction $\overline{T(x,y,z',t)T(x,y,z'',t)}$



Spectra $S_2(\eta)$



Correlation $R_3(\tau)$



Spectra $S_3(\zeta)$

4. Conclusion

We can see both $S_1(\xi)$, $S_2(\eta)$ are recovered satisfactory, while since the value of $R_3(\tau)$ beyond $(-3,3)$ is a trivial $R(\tau)$ divided by a trivial $\overline{T(x,y',z,t)T(x,y'',z,t)}$, and divided by a trivial $\overline{T(x,y,z',t)T(x,y,z'',t)}$ again, it's very difficult to have a genuine correlation result, and hence it's also very difficult to get a genuine $S_3(\zeta)$.